Larmor, Aether and Matter (Adams Prize Essay), presented by Mr. Newall; International Union for Solar Research, Transactions, vol. 2, presented by the publishers; John Tebbutt, Astronomical Memoirs, presented by the author; Photographs of Comet *c* 1908, further series of 15 transparencies, presented by the Astronomer Royal; Photographs of the Moon, etc., taken with a cœlostat reflector, presented by Mr. J. H. Reynolds.

The Relations between Position Angle and Distance and Standard (photographic) Coordinates. By H. C. Plummer, M.A.

1. There is reason to believe that some micrometrical observers recognise some advantage in comparing photographic results with their own differential measures, but are deterred partly by a supposed difficulty in reducing the photographic measures as they appear, for example, in the Astrographic Catalogues, and partly by a distrust in the accuracy of photographic work. It may be well, therefore, to examine what is involved in a careful reduction, and to give a fully-worked example. The value of the photographic catalogues for the purpose has been pointed out, with a number of illustrative results, by Mr. Innes, in a review of Burnham's General Catalogue of Double Stars (*The Observatory*, xxx. p. 452).

2. Let (ξ, η), (ξ_1, η_1) be the standard coordinates of the principal star and companion respectively on a plate whose centre is at R.A. A and decl. D, the focal length being for the present taken as unity. Let the position-angle and distance be p and s. It will be convenient to put

$$N^2 = 1 + \xi^2 + \eta^2; \quad N_1{}^2 = 1 + \xi_1{}^2 + \eta_1{}^2 \quad . \quad . \quad (1)$$

$$\Delta\xi = \xi_1 - \xi, \qquad \Delta\eta = \eta_1 - \eta \quad . \quad . \quad . \quad (2)$$

Then, since $(\xi, \eta, 1)$ and $(\xi_1, \eta_1, 1)$ are proportional to the direction-cosines of lines drawn in the direction of the two stars, we have at once

$$\cos s = (\xi\xi_1 + \eta\eta_1 + 1)/NN_1 \quad . \quad . \quad . \quad (3)$$

which gives

$$\sin^2 s = \{\Delta\xi^2 + \Delta\eta^2 + (\eta\Delta\xi - \xi\Delta\eta)^2\}/N^2N_1{}^2 \, . \quad . \quad (4)$$

Now, on an astrographic plate, ξ and η can scarcely exceed $\frac{1}{60}$, and are usually much less. Hence, if we wish to measure distances up to 300″ and allow an error of 0″·1 in extreme cases on such plates, we can put simply

$$s = (\Delta\xi^2 + \Delta\eta^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (5)$$

On larger plates it may be necessary to use a closer approximation.

3. If the principal stars were at the centre of the plate, ξ_1, η_1 would give immediately p and s. But in any case we can suppose a transformation to a second plate so as to bring the principal star to the centre and the other star to the point (ξ', η'). It will be convenient to put

$$L^2 = \xi^2 + (\cos D - \eta \sin D)^2 \quad . \quad . \quad . \quad (6)$$

Then we shall have

$$\xi' = \tan s \sin p$$

$$= \{(\cos D - \eta \sin D)\Delta\xi + \xi \sin D . \Delta\eta\} N/(\xi\xi_1 + \eta\eta_1 + 1)L \quad . \quad (7)$$

$$\eta' = \tan s \cos p$$

$$= \{ -\xi(\sin D + \eta \cos D)\Delta\xi + [(1 + \xi^2) \cos D - \eta \sin D]\Delta\eta\} \\ /(\xi\xi_1 + \eta\eta_1 + 1)L \quad (8)$$

Exact transformation formulæ in this precise form, involving only ξ, η and D, do not seem to have been given before, and proofs will be found at the end of this paper. Hence, neglecting terms multiplied by ξ^2, $\xi\eta$ and η^2,

$$\tan p = \frac{(\cos D - \eta \sin D)\Delta\xi + \xi \sin D . \Delta\eta}{-\xi \sin D . \Delta\xi + (\cos D - \eta \sin D)\Delta\eta}$$

or

$$p = \tan^{-1}\Delta\xi/\Delta\eta + \tan^{-1}\{\xi \tan D/(1 - \eta \tan D)\} \quad . \quad (9)$$

The terms neglected in (7) and (8) do not increase with D, and therefore (9) holds for small plates in any part of the sky. But, except in the immediate neighbourhood of the pole, a further simplification can be made, and we may write

$$p = \tan^{-1}\Delta\xi/\Delta\eta + \xi \tan D \quad . \quad . \quad . \quad . \quad . \quad (10)$$

4. Thus far it has been assumed that the standard coordinates are referred to unit focal length, *i.e.* are expressed, so to speak, in circular measure. But it is clear that in applying the simple formulæ (5) and (10) this restriction is no longer necessary, and that the standard coordinates may be taken in any definite angular unit. Moreover, the catalogues give measured, not standard coordinates. The total want of uniformity in the presentation of their results by the different observatories is regrettable, and the following remarks will strictly apply only to the Oxford Catalogue. We have

$$\xi = x - Ax - By - C - 13$$
$$\eta = y - Dx - Ey - F - 13$$

the plate-constants being given with the measured coordinates x, y. Hence

$$\Delta\xi = \Delta x - A.\Delta x - B.\Delta y, \quad \Delta\eta = \Delta y - D.\Delta x - E.\Delta y \quad . \quad (11)$$

where $\Delta x = x_1 - x$ and $\Delta y = y_1 - y$ are expressed in the unit $5'$.

The plate-constants are small, so that approximately $\xi = x - 13$ and the equations (5) and (10) may be re-written in the form

$$s = (\Delta\xi^2 + \Delta\eta^2)^{\frac{1}{2}} \,.\, 300'' \quad . \quad . \quad . \quad (12)$$

$$p = \tan^{-1}\Delta\xi/\Delta\eta + 1^\circ.(x - 13)\tan D/12 \quad . \quad (13)$$

s and p being thus expressed, as usual, in seconds and degrees respectively.

5. As an illustration, chosen at random, I have taken the case of the stars in the neighbourhood of υ Coronæ, which is 7566 in the Cambridge A.G. Catalogue, and 7570 in Burnham's General Catalogue. The stars required are easily identified on the plates of two different zones, and the following extracts are set down :—

R.A. $16^h\ 16^m + 29^\circ$.

Plate 1011 ; 1896 April 21.

A.	B.	C.
+ ·00772	+ ·00834	− ·3619
D.	**E.**	**F.**
− ·00807	+ ·00760	− ·1459

Mag. $= 14{\cdot}1 - 1{\cdot}02\sqrt{d}$.

No.	d.	x.	y.
38718*	75	4·304	17·754
719	10	4·385	17·989
729	18	4·416	18·020
730	14	4·634	18·005

R.A. $16^h\ 12^m + 30^\circ$.

Plate 1952 ; 1902 May 27.

A.	B.	C.
− ·00084	+ ·00024	− ·0855
D.	**E.**	**F.**
− ·00002	− ·00064	− ·3676

Mag. $= 16{\cdot}0 - 1{\cdot}25\sqrt{d}$.

No.	d.	x.	y.	Star.
35790*	80	14·843	5·398	A
791	13	14·920	5·635	E
792	23	14·952	5·666	C
793	22	15·167	5·653	D

	AC.	AD.	CE.	AC.	AD.	CE.
Δx	+ ·1120	+ ·3300	− ·0310	+ ·1090	+ ·3240	− 0320
$-$ A. Δx	− 9	− 25	+ 2	+ 1	+ 3	0
$-$ B. Δy	− 22	− 21	+ 3	− 1	− 1	0
$\Delta\xi$	+ ·1089	+ ·3254	− ·0305	+ ·1090	+ ·3242	− ·0320
Δy	+ ·2660	+ ·2510	− ·0310	+ ·2680	+ ·2550	− ·0310
$-$ D. Δx	+ 9	+ 27	− 3	0	0	0
$-$ E. Δy	− 20	− 19	+ 2	+ 2	+ 2	0
$\Delta\eta$	+ ·2649	+ ·2518	− ·0311	+ ·2682	+ ·2552	− ·0310
$\sqrt{(\Delta\xi^2 + \Delta\eta^2)}$	·2864	·4114	·0436	·2895	·4126	·0446
s	85″·92	123″·42	13″·08	86″·85	123″·78	13″·38
$\tan^{-1}\Delta\xi/\Delta\eta$	22°·4	52°·3	224°·4	22°·1	51°·8	225°·9
$(x-13)/12$	−0·72	−0·72	−0·72	+ 0·15	+ 0·15	+0·16
$\times \tan$ D	−0°·4	−0°·4	−0°·4	+0°·1	+0°·1	+0°·1
p	22°·0	51°·9	224°·0	22°·2	51°·9	226°·0

6. These results, which are referred to the equinox 1900·0, can now be compared with the micrometer observations given in Burnham's General Catalogue.

υ Coronæ = Burnham 7570.

p.	s.	Mags.		Ep.	Obs.	
		A and B.				
29°·5	55″·98	3	12·0	1879·32	β	2
28·7	55·45		12·7	1898·01	Doo.	2n
28·7	54·50		13	1905·05	β	1n

There is no image of the star B on either of the Oxford plates. The photographic magnitude is presumably fainter than 12·5.

p.	s.	Mags.		Ep.	Obs.	
		A and C.				
24·5	88·69		(13)	1823·36	Sh	2
22·4	86·31		9·3	1879·32	β	2n
22·0	85·92	5·3	9·8	1896·30	Oxf.	
22·4	85·35		8·5	1897·90	Doo.	5n
22·2	86·85	4·8	10·0	1902·40	Oxf.	
22·1	86·67		10·8	1905·05	β	1n
		A and D.				
54·9	126·42		(12)	1823·36	Sh.	2
52·2	123·61		9·0	1879·32	β	2n
51·9	123·42	5·3	10·3	1896·30	Oxf.	
51·9	123·70		8·0	1897·90	Doo.	5n
51·9	123·78	4·8	10·1	1902·40	Oxf.	
51·5	123·75		10	1905·05	β	1n
		C and E.				
222·7	13·23	9·3	10·5	1879·32	β	2
224·0	13·08	9·8	10·9	1896·30	Oxf.	
226·0	13·38	10·0	11·5	1902·40	Oxf.	
226·0	13·34	10·8	11·5	1905·05	β	1n

7. The evidence given by this group of four stars occurring on two different plates suggests the following inferences:—

(1) Differential measures of pairs of stars can be obtained from the published Astrographic Catalogues with very little trouble.

(2) Such measures have an accuracy of the same class as the results of the very best visual observers.

(3) The photographic magnitudes are, at any rate, not inferior to the estimates of the skilled double-star observer.

(4) If the foregoing conclusions are justified, some guidance may be obtained as regards the policy which the observer at the

telescope should follow. In general, the photographic method does not allow of the accurate measurement of very close companions; the limit of separation required depends on the magnitude of the components. Also the faintest stars lie outside the great mass of easily accessible data which the Astrographic Catalogues will contain. There is thus an ample field in which the work of the visual observer will be complementary to the photographic results. But in future care ought to be exercised not to trench on ground where visual observations will be purely redundant.

8. Proofs will now be given of the transformation formulæ (7) and (8) which were employed in § 3. Let (x, y, z) be the coordinates of a point referred to the rectangular axes OA, OB, OC, where C is the plate centre, R.A. = A and decl. = D, and the plane OCB passes through the pole P. Let also (x', y', z') be the coordinates of the same point referred to the axes OA′, OB′, OC′, where C′ is a new plate centre, R.A. = α and decl. = δ, and the plane OC′ B′ passes through the pole P. The relations between the two systems will be given by a transformation scheme

	x'	y'	z'
x	l_1	m_1	n_1
y	l_2	m_2	n_2
z	l_3	m_3	n_3

Now if (x'', y'', z'') be the coordinates of the same point referred to the axes O♈, OL, OP (γ being the first point of Aries), the relations of these to (x, y, z) are given by

	x''	y''	z''
x	$-\sin A$	$\cos A$	0
y	$-\sin D \cos A$	$-\sin D \sin A$	$\cos D$
z	$\cos D \cos A$	$\cos D \sin A$	$\sin D$

with a similar scheme for x', y', z', in terms of α and δ. Hence, when x'', y'', z'' are eliminated, we find

$$
\begin{aligned}
l_1 &= \cos(\alpha - A) \\
m_1 &= -\sin\delta \sin(\alpha - A) \\
n_1 &= \cos\delta \sin(\alpha - A) \\
l_2 &= \sin D \sin(\alpha - A) \\
m_2 &= \sin\delta \sin D \cos(\alpha - A) + \cos\delta \cos D \\
n_2 &= -\cos\delta \sin D \cos(\alpha - A) + \sin\delta \cos D \\
l_3 &= -\cos D \sin(\alpha - A) \\
m_3 &= -\sin\delta \cos D \cos(\alpha - A) + \cos\delta \sin D \\
n_3 &= \cos\delta \cos D \cos(\alpha - A) + \sin\delta \sin D
\end{aligned}
$$

Now ξ, η being the coordinates of C′ on the plate with centre C,

$$
\begin{aligned}
N \cos\delta \sin(\alpha - A) &= \xi \\
N \sin\delta &= \sin D + \eta \cos D \\
N \cos\delta \cos(\alpha - A) &= \cos D - \eta \sin D
\end{aligned}
$$

so that

$$N \cos\delta = L,$$

where N and L have the meanings defined by (1) and (6). Hence we easily derive

$$\begin{aligned}
l_1 &= (\cos D - \eta \sin D)/L \\
m_1 &= -\xi(\sin D + \eta \cos D)/LN \\
n_1 &= \xi/N \\
l_2 &= \xi \sin D/L \\
m_2 &= \{(1+\xi^2)\cos D - \eta \sin D\}/LN \\
n_2 &= \eta/N \\
l_3 &= -\xi \cos D/L \\
m_3 &= \{(\xi^2+\eta^2)\sin D - \eta \cos D\}/LN \\
n_3 &= 1/N
\end{aligned}$$

But since ξ', η' and ξ_1, η_1 are corresponding coordinates on the plates whose centres are C′ and C respectively,

$$\begin{aligned}
\xi' &= x'/z' = (l_1\xi_1 + l_2\eta_1 + l_3)/(n_1\xi_1 + n_2\eta_1 + n_3) \\
\eta' &= y'/z' = (m_1\xi_1 + m_2\eta_1 + m_3)/(n_1\xi_1 + n_2\eta_1 + n_3)
\end{aligned}$$

and therefore

$$\xi' = \{(\cos D - \eta \sin D)\,\xi_1 + \xi \sin D.\eta_1 - \xi \cos D\}N/(\xi\xi_1 + \eta\eta_1 + 1)L$$

$$\eta' = [-\xi(\sin D + \eta \cos D)\xi_1 + \{(1+\xi^2)\cos D - \eta \sin D\}\,\eta_1 + (\xi^2+\eta^2)\sin D - \eta \cos D]/(\xi\xi_1 + \eta\eta_1 + 1)L$$

which by (2) become identical with (7) and (8).

9. Similar expressions may be obtained in another way. Let S_0 be the principal star and S_1 the second star. With respect to the plate centre C let p_0, s_0 be the position-angle and distance of S_0, p_1 and s_1 of S_1, so that

$$\xi = \tan s_0 \sin p_0\,,\quad \eta = \tan s_0 \cos p_0\,,\quad \cos s_0 = 1/N$$

$$\xi_1 = \tan s_1 \sin p_1\,,\quad \eta_1 = \tan s_1 \cos p_1\,,\quad \cos s_1 = 1/N_1$$

Then

$$\begin{aligned}
\sin PS_0 \sin s \sin p &= \sin PS_0 \sin PS_1 \sin (CPS_1 - CPS_0) \\
&= \sin s_1 \sin p_1 (\cos D \cos s_0 - \sin D \sin s_0 \cos p_0) \\
&\quad - \sin s_0 \sin p_0 (\cos D \cos s_1 - \sin D \sin s_1 \cos p_1) \\
&= \{\xi_1(\cos D - \eta \sin D) - \xi(\cos D - \eta_1 \sin D)\}/NN_1 \\
\sin PS_0 \sin s \cos p &= \cos PS_1 - \cos S_0P \cos S_0S_1 \\
&= \sin D \cos s_1 + \cos D \sin s_1 \cos p_1 \\
&\quad - (\sin D \cos s_0 + \cos D \sin s_0 \cos p_0). \\
&\qquad \{\cos s_1 \cos s_0 + \sin s_1 \sin s_0 \cos (p_0 - p_1)\} \\
&= (\sin D + \eta_1 \cos D)/N_1 \\
&\quad - (\sin D + \eta \cos D)(1 + \xi\xi_1 + \eta\eta_1)/N^2N_1 \\
&= [\sin D\ (\xi^2 + \eta^2 - \xi\xi_1 - \eta\eta_1) \\
&\quad + \cos D\{\eta_1 - \eta + \xi(\xi\eta_1 - \xi_1\eta)\}]/N^2N_1 \\
\sin^2 PS_0 &= 1 - (\sin D \cos s_0 + \cos D \sin s_0 \cos p_0)^2 \\
&= 1 - (\sin D + \eta \cos D)^2/N^2 = L^2/N^2
\end{aligned}$$

Hence we have

$$\sin s \sin p = \{\cos D.\Delta\xi - \sin D(\eta\Delta\xi - \xi\Delta\eta)\}/LN_1$$

$$\sin s \cos p = [-\sin D(\xi\Delta\xi + \eta\Delta\eta) + \cos D\{\Delta\eta - \xi(\eta\Delta\xi - \xi\Delta\eta)\}]/LNN_1$$

which, when (3) is remembered, are easily seen to be equivalent to (7) and (8).

University Observatory, Oxford:
1908 *December* 10.

Analysis of the Colours and Magnitudes of 3630 *Stars between the N. Pole and* 25° *S. Declination.* By W. S. Franks.

During the recent investigation on star colours and spectra (*Monthly Notices*, lxviii. 672), it occurred to me that an analysis of the colours and magnitudes might form a fitting sequel, and might possibly furnish some additional information. I therefore tabulated each hour of R.A. separately into 6 magnitude groups and 7 colour groups (as before); the magnitudes were all based on the *Revised Harvard Photometry*, and the minimum limit was taken at 6·5 of that scale. Of course this list does not contain *all* the 6·5 magnitudes to be found in the *R.H.P.*, but only those observed. The *comites* of double stars were carefully excluded. It is unnecessary waste of space to give the extended details here; suffice it to say that they were all ultimately arranged in four divisions—two galactic (hours III to VIII, and XV to XX); and two non-galactic (hours XXI to II, and IX to XIV). These were again combined into two—galactic and non-galactic—as here shown:—

A. *Combined galactic regions.*

Magnitude.	White.	Yellowish-White.	Pale Yellow.	Yellow.	Pale Orange.	Orange.	Orange-Red.	Total.
>1·5	4	3	1	1	...	1	1	11
1·6–2·5	14	1	1	1	...	...	...	17
2·6–3·5	24	9	15	19	...	2	...	69
3·6–4·5	90	42	45	42	9	4	...	232
4·6–5·5	268	166	139	84	48	27	1	733
5·6–6·5	292	168	208	82	82	51	10	893
Total	692	389	409	229	139	85	12	1955